ICSV15
15th International Congress on Sound and Vibration
6-10 July 2008, Daejeon, Korea

COUPLED THERMOELASTIC VIBRATION ANALYSIS OF BEAMS BASED ON THIRD-ORDER SHEAR DEFORMATION THEORY

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Abstract

This paper presents an analytical solution using the finite Fourier transformation for coupled thermoelastic vibration analysis of a beam based on the third-order shear deformation theory subjected to thermal shock loads. The beam is made of homogenous and isotropic materials. The equation of motion and the conventional coupled energy equation are simultaneously solved to obtain the displacement components and temperature distribution in the beam. Results are presented for simply supported boundary conditions and are verified with those reported in the literature.

1. INTRODUCTION

McQuillen and Brull [1] presented analytical solution for the dynamic, thermoelastic response of cylindrical shells using a variational theorem. Coupled thermoelasticity of beams is discussed by Massalas and Kalpakidis [2,3]. The analytical solution of the coupled thermoelasticity of beams with the Euler-Bernoulli assumption is given in [2], and that with Timoshenko assumption is given in [3]. In the treatment of these problems a linear approximation for temperature variation across the thickness direction of the beam is considered. Eslami and Vahedi [4] presented the one-dimensional coupled thermoelasticity problem of rods using the Galerkin finite element method. Finite element coupled thermoelastic analysis of composite Timoshenko-beams is given by Maruthi and Sinha [5], where the temperature variation across the thickness direction is neglected. Manoach and Ribeiro developed a numerical procedure to study the coupled large amplitude thermoelastic vibrations of Timoshenko beams subjected to the thermal and mechanical loads using the finite difference approximation and modal coordinate transformations [6]. The thermoelastic damping of micro-beam is analyzed by Sun and et al. [7] using both the finite Fourier transformation method combined with Laplace transformation and the normal mode analysis.

This paper presents the behavior of a beam under lateral thermal shock with coupled thermoelastic assumption. The analysis is based on the finite Fourier transformation method. The beam formulations are based on the third-order shear deformation theory.

2. DERIVATION OF GOVERNING EQUATIONS

Consider a beam of rectangular cross section with height l, height h and width b, as shown in Figure 1.

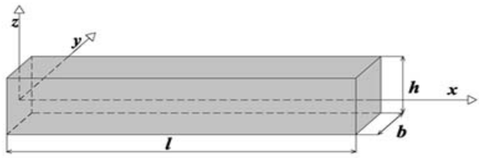

Figure 1. The beam and coordinates

Using the third-order shear deformation theory, the displacement components are [8]

$$u(x,z,t) = u_0(x,t) + z\psi(x,t) - c'z^3(w_{,x} + \psi),$$
$$w(x,t) = w(x,t). \tag{1}$$

where u is the axial displacement, w is the transverse displacement in the z direction and ψ is the rotation angle of the cross-section with respect to the longitudinal axis. The constant c' is given by $c'=4/3h^2$.The subscript zero denotes middle surface and a comma denotes partial differentiation. In terms of the displacement components, the normal and shear strains are given by

$$\varepsilon_x = u_{0,x} + z\psi_{,x} - c'z^3(w_{,xx} + \psi_{,x}),$$
$$\gamma_{xz} = (w_{,x} + \psi) - 3c'z^2(w_{,x} + \psi). \tag{2}$$

Assuming that the beam material is linear elastic and isotropic, the stress-displacement relations for the beam are

$$\sigma_x = E[u_{0,x} + z\psi_{,x} - c'z^3(\psi_{,x} + w_{,xx})] - E\alpha\theta,$$
$$\sigma_{xz} = G[\psi + w_{,x} - 3c'z^2(\psi + w_{,x})]. \tag{3}$$

where G is the shear modulus, α is the coefficient of thermal expansion, $\theta=T-T_0$ is the temperature change, and T_0 is the reference temperature, respectively.

We assumed that the temperature change along the height direction is linear. This assumption is justified considering that the thickness of the beam is small with respect to its length [2, 3]:

$$\theta = \theta_1(x,t) + \frac{z}{h}\theta_2(x,t). \tag{4}$$

where θ_1 and θ_2 are unknown to be found through the solution of the coupled equations.

2.1 Equation of Motion

The equations of motion of a beam based on the third-order shear deformation theory is [8]

$$N_{,x} = I_0\,u_{0,tt} + J_1\psi_{,tt} - c'I_3\,w_{,xtt}$$
$$\overline{Q}_{,x} + c'P_{,xx} = I_0w_{,tt} + c'I_3u_{0,xtt} + J_4\psi_{,xtt} - c'^2I_6w_{,xxtt}$$
$$\overline{M}_{,x} - \overline{Q} = J_1u_{0,tt} + K_2\psi_{,tt} - J_4w_{,xtt} \tag{5}$$

where

$$N = \int_z \sigma_x dz,\ M = \int_z \sigma_x z\, dz,\ Q = \int_z \sigma_{xz} dz,\ P = \int_z \sigma_x z^3\, dz,\ R = \int_z \sigma_{xz} z^2 dz.$$
$$\overline{Q} = Q - 3c'R,\ \overline{M} = M - c'P.\ (I_i) = \int_z \rho z^i dz \quad i = 0,1,\ldots,6 \tag{6}$$
$$J_i = I_i - c_1 I_{i+2},\ K_2 = I_2 - 2c_1 I_4 + c_1^{\,2} I_6.$$

where ρ is the mass density of the beam.

Substituting Eqs. (3), (4), and (6) into Eqs. (5), the equations of motion become

$$A_1 u_{0,xx} + A_2 \psi_{,xx} + A_3 w_{,xxx} + A_4 \theta_{1,x} + A_5 \theta_{2,x} + A_6 u_{0,tt} + A_7 \psi_{,tt} + A_8 w_{,xtt} = 0,$$
$$B_1 u_{0,xxx} + B_2 \psi_{,x} + B_3 \psi_{,xxx} + B_4 w_{,xx} + B_5 w_{,xxxx} + B_6 \theta_{1,xx} + B_7 \theta_{2,xx} + B_8 u_{0,xtt} +$$
$$B_9 \psi_{,xtt} + B_{10} w_{,tt} + B_{11} w_{,xxtt} = 0, \tag{7}$$
$$C_1 u_{0,xx} + C_2 \psi + C_3 \psi_{,xx} + C_4 w_{,x} + C_5 w_{,xxx} + C_6 \theta_{1,x} + C_7 \theta_{2,x} + C_8 u_{0,tt} + C_9 \psi_{,tt}$$
$$+ C_{10} w_{,xtt} = 0.$$

where A's, B's and C's are the constants of Eqs. (7).

Simply supported boundary conditions are considered for the beam and the beam is assumed to be initially at zero deflection

$$u_{0,x}(0,t) = u_{0,x}(l,t) = 0,\ \&\ \psi_{,x}(0,t) = \psi_{,x}(l,t) = 0,\ \&\ w(0,t) = w(l,t) = 0, \qquad t > 0$$
$$u_0(x,0) = \psi(x,0) = w(x,0) = 0.\ \ 0 \le x \le l \tag{8}$$

2.1 Energy Equation

The first law of thermodynamics for heat conduction in beam in the coupled form is [4]

$$\left(k\theta_{,i}\right)_{,i} - \rho c_v \theta_{,t} - \alpha T_0 (3\lambda + 2\mu)\left(\varepsilon_{ii}\right)_{,t} = 0 \tag{9}$$

where k, c_v, α, and ε_{ii} are the thermal conductivity, specific heat, coefficient of linear thermal expansion, and normal strain, respectively, and λ and μ are the Lamé constants.

The energy equation for the beam based on third-order shear deformation theory is reduced to

$$\mathrm{Re}s = k\theta_{,xx} + k\theta_{,zz} - \rho c_v \theta_{,t} - E\alpha T_0 \left(u_{0,xt} + z\psi_{,xt} - c'z^3\left(\psi_{,xt} + w_{,xxt}\right)\right) = 0 \tag{10}$$

The thermal boundary conditions may be assumed that the lower beam surface is heat isolated and a heat flux *q(x,t)* is applied on the upper beam surface. The beam is initially assumed to be at ambient temperature and the thermal boundary and initial conditions are assumed as

$$\theta(0,t) = \theta(l,t) = 0, \qquad t > 0\ \ \&\ \ \theta(x,0) = 0. \qquad 0 \le x \le l \tag{11}$$

Using Eq. (4) and multiplying Eq. (10) by *dz* and *zdz*, integrating over height *h,* the residue *Res* of the energy equation may be made orthogonal with respect to *dz* and *zdz*, to provide two independent equations for two independent functions θ_1 and θ_2 as

$$D_1 \theta_{1,xx} + D_2 \theta_{2,xx} + D_3 \theta_{1,t} + D_4 \theta_{2,t} + D_5 u_{0,xt} + D_6 \psi_{,xt} + D_7 w_{,xxt} + D_8 q^-(x,t) + D_9 q^+(x,t) = 0,$$

$$E_1\theta_{1,xx} + E_2\theta_{2,xx} + E_3\theta_2 + E_4\theta_{1,t} + E_5\theta_{2,t} + E_6 u_{0,xt} + E_7\psi_{,xt} + E_8 w_{,xxt} + E_9 q^-(x,t) + E_{10} q^+(x,t) = 0. \tag{12}$$

where q^+ and q^- are the applied heat flux on the upper surface and lower surface of the beam, respectively. *D*'s and *E*'s are the constants of Eqs. (12).

3. SOLUTION PROCEDURE

To solve the simultaneous governing equations, dimensionless values are defined as

$$\bar{u}_0 = \frac{k}{q_{avrg}\alpha l^2} u_0, \bar{\psi} = \frac{k}{q_{avrg}\alpha l}\psi, \bar{w} = \frac{k}{q_{avrg}\alpha l^2} w, \bar{\theta} = \frac{k}{q_{avrg}\alpha l T_0}\theta, \bar{x} = \frac{x}{l},\ \bar{t} = \frac{\kappa t}{h^2}. \tag{13}$$

where q_{avrg} and κ are the average of heat flux and thermal diffusivity, respectively. The bar values indicate dimensionless parameters.

Assuming Eqs. (7) and (12) and using the dimensionless parameters, the five coupled governing equations are

$$\begin{aligned}
&a_1\bar{u}_{0,\bar{x}\bar{x}} + a_2\bar{\psi}_{,\bar{x}\bar{x}} + a_3\bar{w}_{,\bar{x}\bar{x}\bar{x}} + a_4\bar{\theta}_{1,\bar{x}} + a_5\bar{\theta}_{2,\bar{x}} + a_6\bar{u}_{0,\bar{t}\bar{t}} + a_7\bar{\psi}_{,\bar{t}\bar{t}} + a_8\bar{w}_{,\bar{x}\bar{t}\bar{t}} = 0,\\
&b_1\bar{u}_{0,\bar{x}\bar{x}\bar{x}} + b_2\bar{\psi}_{,\bar{x}} + b_3\bar{\psi}_{,\bar{x}\bar{x}\bar{x}} + b_4\bar{w}_{,\bar{x}\bar{x}} + b_5\bar{w}_{,\bar{x}\bar{x}\bar{x}\bar{x}} + b_6\bar{\theta}_{1,\bar{x}\bar{x}} + b_7\bar{\theta}_{2,\bar{x}\bar{x}} + b_8\bar{u}_{0,\bar{x}\bar{t}\bar{t}}\\
&+ b_9\bar{\psi}_{,\bar{x}\bar{t}\bar{t}} + b_{10}\bar{w}_{,\bar{t}\bar{t}} + b_{11}\bar{w}_{,\bar{x}\bar{x}\bar{t}\bar{t}} = 0,\\
&c_1\bar{u}_{0,\bar{x}\bar{x}} + c_2\bar{\psi} + c_3\bar{\psi}_{,\bar{x}\bar{x}} + c_4\bar{w}_{,\bar{x}} + c_5\bar{w}_{,\bar{x}\bar{x}\bar{x}} + c_6\bar{\theta}_{1,\bar{x}} + c_7\bar{\theta}_{2,\bar{x}} + c_8\bar{u}_{0,\bar{t}\bar{t}} + c_9\bar{\psi}_{,\bar{t}\bar{t}} + c_{10}\bar{w}_{,\bar{x}\bar{t}\bar{t}} = 0,\\
&d_1\bar{\theta}_{1,\bar{x}\bar{x}} + d_2\bar{\theta}_{2,\bar{x}\bar{x}} + d_3\bar{\theta}_{1,\bar{t}} + d_4\bar{\theta}_{2,\bar{t}} + d_5\bar{u}_{0,\bar{x}\bar{t}} + d_6\bar{\psi}_{,\bar{x}\bar{t}} + d_7\bar{w}_{,\bar{x}\bar{x}\bar{t}} + d_8 q^- + d_9 q^+ = 0,\\
&e_1\bar{\theta}_{1,\bar{x}\bar{x}} + e_2\bar{\theta}_{2,\bar{x}\bar{x}} + e_3\bar{\theta}_2 + e_4\bar{\theta}_{1,\bar{t}} + e_5\bar{\theta}_{2,\bar{t}} + e_6\bar{u}_{0,\bar{x}\bar{t}} + e_7\bar{\psi}_{,\bar{x}\bar{t}} + e_8\bar{w}_{,\bar{x}\bar{x}\bar{t}} + e_9 q^- + e_{10} q^+ = 0.
\end{aligned} \tag{14}$$

where *a*'s, *b*'s, *c*'s, *d*'s and *e*'s are dimensionless constants of coupled equations. Simultaneous solution of these equations provides the distribution of the displacement components of the beam and the temperature variables.

Regarding the boundary conditions in Eqs. (8) and (11), to solve the system of motion and energy equations a finite Fourier transformation can be used as [2,3]

$$\begin{aligned}
&\bar{u}_{0m}(m,\bar{t}) = \int_0^1 \bar{u}_0(\bar{x},\bar{t})\cos(m\pi\bar{x})d\bar{x},\ \bar{\psi}_m(m,\bar{t}) = \int_0^1 \bar{\psi}(\bar{x},\bar{t})\cos(m\pi\bar{x})d\bar{x},\\
&\bar{w}_m(m,\bar{t}) = \int_0^1 \bar{w}(\bar{x},\bar{t})\sin(m\pi\bar{x})d\bar{x},\\
&\bar{\theta}_{1m}(m,\bar{t}) = \int_0^1 \bar{\theta}_1(\bar{x},\bar{t})\sin(m\pi\bar{x})d\bar{x},\ \bar{\theta}_{2m}(m,\bar{t}) = \int_0^1 \bar{\theta}_2(\bar{x},\bar{t})\sin(m\pi\bar{x})d\bar{x}.
\end{aligned} \tag{15}$$

where $m=1,3,5,\ldots$.

The solution of Eqs. (15) automatically satisfy to the boundary conditions (i.e., Eqs. (8) and (11)). Based on the Fourier series theory, the inverse transformation towards Eqs. (16) can be expressed by

$$\begin{aligned}
&\bar{u}_0(\bar{x},\bar{t}) = 2\sum_m \bar{u}_{0m}(m,\bar{t})\cos(m\pi\bar{x}),\ \bar{\psi}(\bar{x},\bar{t}) = 2\sum_m \bar{\psi}_m(m,\bar{t})\cos(m\pi\bar{x}),\\
&\bar{w}(\bar{x},\bar{t}) = 2\sum_m \bar{w}_m(m,\bar{t})\sin(m\pi\bar{x}),
\end{aligned}$$

$$\bar{\theta}_1(\bar{x},\bar{t}) = 2\sum_m \bar{\theta}_{1m}(m,\bar{t})\sin(m\pi\bar{x}),\ \bar{\theta}_2(\bar{x},\bar{t}) = 2\sum_m \bar{\theta}_{2m}(m,\bar{t})\sin(m\pi\bar{x}).m = 1,3,... \tag{16}$$

Applying transformations (i.e., Eqs. (15)). to Eqs. (14) and the initial conditions Eqs. (8) and (11) leads to

$$\begin{aligned}
&-r^2 a_1\bar{u}_{0m} - r^2 a_2\bar{\psi}_m - r^3 a_3\bar{w}_m + ra_4\bar{\theta}_{1m} + ra_5\bar{\theta}_{2m} + a_6\bar{u}_{0m,\bar{t}\bar{t}} + a_7\bar{\psi}_{m,\bar{t}\bar{t}} + a_8\bar{w}_{m,\bar{t}\bar{t}} = 0,\\
&r^3 b_1\bar{u}_{0m} - rb_2\bar{\psi}_m + r^3 b_3\bar{\psi}_m - r^2 b_4\bar{w}_m + r^4 b_5\bar{w}_m - r^2 b_6\bar{\theta}_{1m} - r^2 b_7\bar{\theta}_{2m} - rb_8\bar{u}_{0m,\bar{t}\bar{t}} -\\
&rb_9\bar{\psi}_{m,\bar{t}\bar{t}} + b_{10}\bar{w}_{m,\bar{t}\bar{t}} - r^2 b_{11}\bar{w}_{m,\bar{t}\bar{t}} = 0,\\
&-r^2 c_1\bar{u}_{0m} + c_2\bar{\psi}_m - r^2 c_3\bar{\psi}_m + rc_4\bar{w}_m - r^3 c_5\bar{w}_m + rc_6\bar{\theta}_{1m} + rc_7\bar{\theta}_{2m} + c_8\bar{u}_{0m,\bar{t}\bar{t}} + c_9\bar{\psi}_{m,\bar{t}\bar{t}}\\
&+ rc_{10}\bar{w}_{m,\bar{t}\bar{t}} = 0,\\
&-r^2 d_1\bar{\theta}_{1m} - r^2 d_2\bar{\theta}_{2m} + d_3\bar{\theta}_{1m,\bar{t}} + d_4\bar{\theta}_{2m,\bar{t}} - rd_5\bar{u}_{0m,\bar{t}} - rd_6\bar{\psi}_{m,\bar{t}} - r^2 d_7\bar{w}_{m,\bar{t}} + (2d_8/r)q^-\\
&+ (2d_9/r)q^+ = 0,\\
&-r^2 e_1\bar{\theta}_{1m} - r^2 e_2\bar{\theta}_{2m} + e_3\bar{\theta}_{2m} + e_4\bar{\theta}_{1m,\bar{t}} + e_5\bar{\theta}_{2m,\bar{t}} - re_6\bar{u}_{0m,\bar{t}} - re_7\bar{\psi}_{m,\bar{t}} - r^2 e_8\bar{w}_{m,\bar{t}} +\\
&(2e_9/r)q^- + (2e_{10}/r)q^+ = 0. \quad r = m\pi
\end{aligned} \tag{17}$$

3.1 Laplace Transform

The system of coupled Eqs. (17) are functions of the Fourier parameter *m* and time *t*. The solution presented in this paper is obtained by finite Fourier transformation, where time is eliminated using the Laplace transform. Once the solution in space domain is obtained, an analytical scheme is used for the Laplace transform to find the final solution in real time domain. Assuming the lower beam surface is heat isolated (q^-=0) and a step function heat flux q^+ is applied on the upper beam surface and applying the Laplace transform to Eqs. (17) give

$$\begin{aligned}
&-r^2 a_1 U_{0m} - r^2 a_2\Psi_m - r^3 a_3 W_m + ra_4\Theta_{1m} + ra_5\Theta_{2m} + a_6 s^2 U_{0m} + a_7 s^2\Psi_m + a_8 s^2 W_m = 0,\\
&r^3 b_1 U_{0m} - rb_2\Psi_m + r^3 b_3\Psi_m - r^2 b_4 W_m + r^4 b_5 W_m - r^2 b_6\Theta_{1m} - r^2 b_7\Theta_{2m} - rb_8 s^2 U_{0m} -\\
&rb_9 s^2\Psi_m + b_{10}s^2 W_m - r^2 b_{11}s^2 W_m = 0,\\
&-r^2 c_1 U_{0m} + c_2\Psi_m - r^2 c_3\Psi_m + rc_4 W_m - r^3 c_5 W_m + rc_6\Theta_{1m} + rc_7\Theta_{2m} + c_8 s^2 U_{0m} + c_9 s^2\Psi_m\\
&+ c_{10}s^2 W_m = 0,\\
&-r^2 d_1\Theta_{1m} - r^2 d_2\Theta_{2m} + d_3 s\Theta_{1m} + d_4 s\Theta_{2m} - rd_5 sU_{0m} - rd_6 s\Psi_m - r^2 d_7 sW_m +\\
&(2d_9/rs)q^+ = 0,\\
&-r^2 e_1\Theta_{1m} - r^2 e_2\Theta_{2m} + e_3\Theta_{2m} + e_4 s\Theta_{1m} + e_5 s\Theta_{2m} - re_6 sU_{0m} - re_7 s\Psi_m - r^2 e_8 sW_m +\\
&(2e_{10}/rs)q^+ = 0.
\end{aligned} \tag{18}$$

where *s* is the Laplace transform parameter and

$$U_{0m} = L[\bar{u}_{0m}], \Psi_m = L[\bar{\psi}_m], W_m = L[\bar{w}_m], \Theta_{1m} = L[\bar{\theta}_{1m}], \Theta_{2m} = L[\bar{\theta}_{2m}]. \tag{19}$$

Thus, the solution of unknown variables in Eqs. (18) in the Laplace transformation domain can be given as

$$F_m(s) = \frac{Q_m(s)}{P_m(s)}. \tag{20}$$

where $Q_m(s)$ and $P_m(s)$ are polynomial functions of s.

After taking the analytical inverse Laplace transform to Eq. (20) [2,3], the solution of unknown variables in time domain are obtained as

$$f_m(t) = \sum_{j=1}^{n_p} \frac{Q_m(s_{p_j})}{P'_m(s_{p_j})} e^{s_{pj}t} \tag{21}$$

where s_{pj} are the roots of $P_m(s)$ and n_p are the number of them. Also, (') sign in superscript indicates derivative respect to s.

4. RESULTS DISCUSSION

To study of lateral thermal shock effect on the beam with coupled thermoelastic assumption, an aluminum beam of length 0.25m and height 0.0022m is assumed. The thermal boundary conditions at the ends of the beam are assumed to be ambient temperature $T_0 = 293K$. The upper side of the beam is subjected to a step function thermal shock while the lower side is insulated.

Figure 2 shows temperature change history between upper and lower surfaces at the midpoint of the heated beam for the uncoupled assumption reported by [2] for Euler-Bernoulli beam and present study. Due to the applied step function thermal shock, the beam temperature peaks to a maximum value, and then diffuses during the time. Close agreement is observed between the two studied in temperature histories. Figure 3 shows the midpoint lateral deflection history for the uncoupled assumptions reported by [2] for Euler-Bernoulli beam and present study. Due to the applied thermal shock, beam vibrated. It is observed that the maximum lateral deflection and the amplitude of vibration, for Euler-Bernoulli beam theory, are greater than that one obtained by using the third-order shear deformation theory.

The temperature change history between upper and lower surfaces at the midpoint of the heated beam for the uncoupled and coupled assumptions is shown in Figure 4. It can be seen

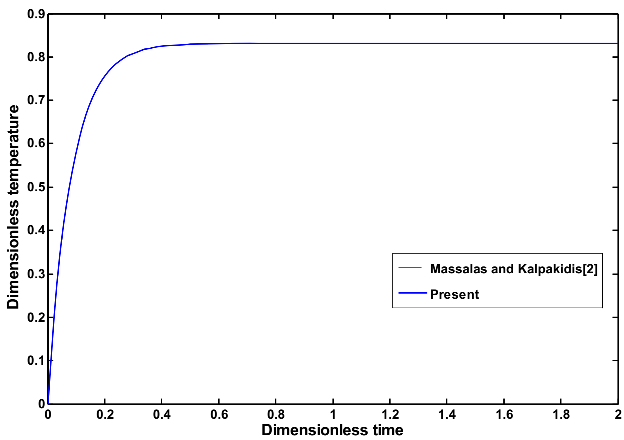

Figure 2. Temperature change history between upper and lower surfaces at the midpoint of the beam.

from Figure 4 that the temperature is vibrating with small amplitude when the coupling between the strain and temperature fields is taken into account. Figure 5 shows the midpoint lateral deflection history for the uncoupled and coupled assumptions. It can be seen from Figure 5 that the vibration of the beam decays with time increasing when the coupling between the strain and temperature fields is taken into account. Also, no significant difference is seen in both coupled and uncoupled solutions. Since the vibrations of the beam for the uncoupled and coupled solutions is hardly distinguished form the curves in Figure 5, the difference between two solutions is shown for long time in Figure 6 to see the vibration decay caused by thermoelastic damping and the couple effect.

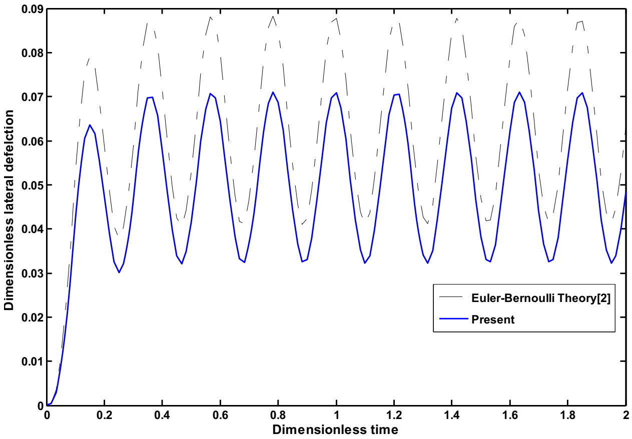

Figure 3. Lateral deflection history at the midpoint of the beam.

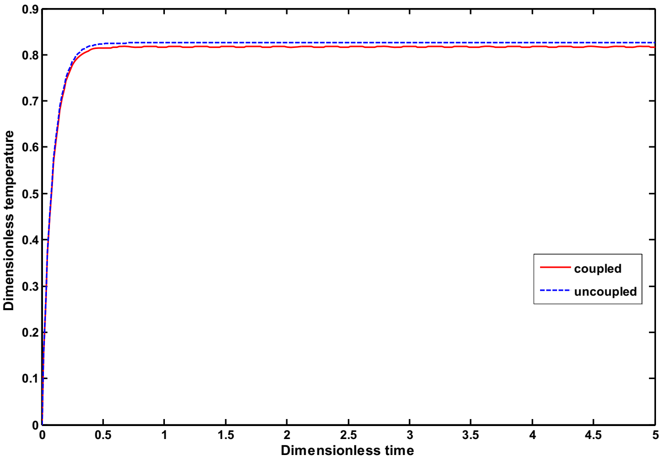

Figure 4. Temperature change history between upper and lower surfaces at the midpoint of the beam for coupled and uncoupled solutions.

5. CONCLUSIONS

In the present paper, the coupled thermoelastic vibration analysis of a beam based on the third-order shear deformation theory is investigated. The beam is subjected to a thermal shock of step function on the upper side while lower side is insulated. Boundary conditions of the beam are taken to be simply supported and have ambient temperature at the ends of the beam.
To solve the problem, the finite Fourier transformation is used. More ever, to treat the time dependency, the Laplace transform technique is applied. The inverse Laplace transform is carried out analytically.
Results show the temperature peaks to a maximum value, and then diffuses to this value. Temperature is vibrating with small amplitude because of coupling effect. It can be said that the maximum lateral deflection and the amplitude of vibration, for Euler-Bernoulli beam theory, are greater than that one obtained by using the third-order shear deformation theory. More ever, generally it can be said that there is no significant difference between coupled and uncoupled solution. However, the effect of coupling is like damping. The vibration of the beam decays with time increasing when the coupling between the strain and temperature fields is taken into account.

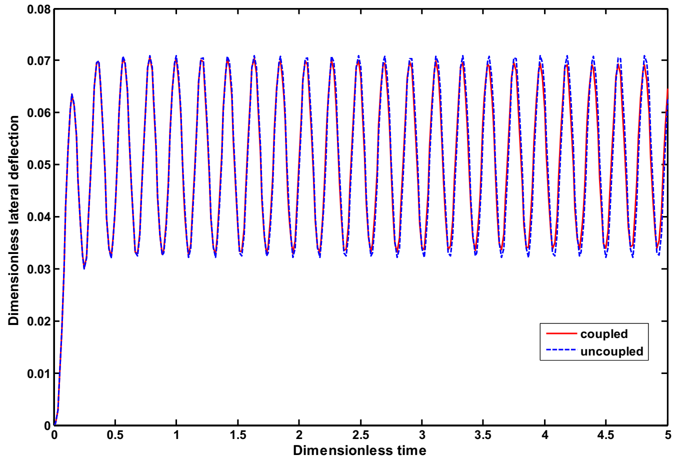

Figure 5. Lateral deflection history at the midpoint of the beam for coupled and uncoupled solutions.

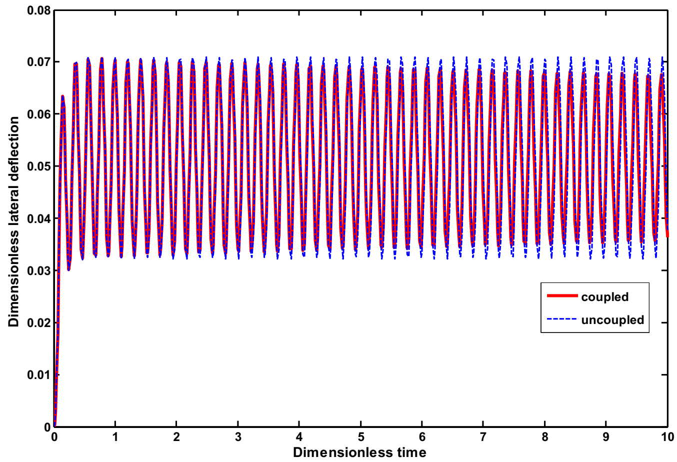

Figure 6. Lateral deflection history at the midpoint of the beam for coupled and uncoupled solutions.

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